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DETERMINING THE CP VIOLATING PHASE γ ^a

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We review several methods for determining the Kobayashi-Maskawa phase $\gamma = \text{Arg}V_{ub}^*$ from rate and CP asymmetry measurements in hadronic B decays. We focus on the processes $B \rightarrow DK$, $B_s \rightarrow D_s K$, and on charmless decays to two light pseudoscalars and decays to a pair of a pseudoscalar and a vector meson. Theoretical uncertainties underlying these methods are discussed.

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1 Introduction

B meson decays open the window into new phenomena of CP nonconservation, providing useful information about CP violating phases¹. Phase measurements for α , β and γ , which are not independent in the Kobayashi-Maskawa framework, are important for two reasons. First, they improve to a higher precision our knowledge of the CKM mixing matrix, consisting of fundamental quark couplings in our present theory. And second, they are sensitive probes for sources of CP violation outside the CKM matrix, and for new flavor changing interactions contributing to $B^0 - \bar{B}^0$ mixing and to rare B decays.

The phase $\beta \equiv \text{Arg}(-V_{cb}^* V_{cd}/V_{tb}^* V_{td})$ ($= \text{Arg} V_{td}^*$ in the standard phase convention) is measured by the time-dependent CP asymmetry in $B^0(t) \rightarrow \psi K_S$ ². Theoretically, this measurement provides a very clean determination of $\sin(2\beta)$, since the single phase approximation holds in this case to better than 1%^{3,4}. The recent CDF measurement⁵, $\sin(2\beta) = 0.79_{-0.44}^{+0.41}$, is an encouraging proof of the method, although it does not yet constitute definite evidence for CP violation in B decays, as predicted in the CKM framework.

The determination of $\sin 2\alpha$ ($\alpha \equiv \text{Arg}(-V_{tb}^* V_{td}/V_{ub}^* V_{ud})$) in $B^0 \rightarrow \pi^+ \pi^-$ suffers from the appearance of a second amplitude (P) due to QCD penguin operators^{3,4}. The weak phase⁶ of P differs from the phase of the dominant tree amplitude (T). The ratio $|P|/|T| = 0.3 \pm 0.1$, obtained⁷ by comparing within broken flavor SU(3) the measured rates⁸ of $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$, implies a potentially large deviation of the measured asymmetry from $\sin 2\alpha$. In fact, the time-dependent asymmetry in $B^0(t) \rightarrow \pi^+ \pi^-$ contains two terms

$$A(t) = a_{\text{dir}} \cos(\Delta mt) + \sqrt{1 - a_{\text{dir}}^2} \sin 2(\alpha + \theta) \sin(\Delta mt), \quad (1)$$

where a_{dir} and θ are both proportional to $|P|/|T|$ and are functions of the relative strong phase between P and T . A precise knowledge of $|P|/|T|$ would permit an accurate determination of α from a measurement of the two terms in the asymmetry^{3,9}. One way of calculating $|P|/|T|$ in perturbative QCD is described by Deshpande at this meeting¹⁰. However, the uncertainty in this calculation has not yet been shown to be under control.

As long as $|P|/|T|$ cannot be calculated reliably, a theoretically cleaner way of resolving the penguin correction will require combining the asymmetry in $B^0 \rightarrow \pi^+ \pi^-$ with other measurements. A very early suggestion¹¹ was to also measure the rates of isospin related processes, $B^+ \rightarrow \pi^+ \pi^0$ and $B^0/\bar{B}^0 \rightarrow \pi^0 \pi^0$. However, it is expected that the color-suppressed rates of tagged B^0 and \bar{B}^0 decays to neutral pions will involve large experimental errors, at least in the first round of experiments. Alternatively, α can be determined by applying the isospin technique to $B \rightarrow \rho\pi$ decays¹², or by relating the time-dependence

in $B^0(t) \rightarrow \pi^+\pi^-$ and in $B_s(t) \rightarrow K^+K^-$ using flavor U-spin symmetry¹³. Since none of these methods is expected to be both theoretically clean and experimentally accessible in the near future, some of these methods will have to be combined in order to reduce the error in α .

The phase $\gamma \equiv \text{Arg}(-V_{ub}^*V_{ud}/V_{cb}^*V_{cd})$ ($= \text{Arg}V_{ub}^*$ in the standard convention) is the relative weak phase between a CKM-favored ($b \rightarrow c$) and a CKM-suppressed ($b \rightarrow u$) decay amplitude. Therefore, it contributes to direct CP asymmetries, in which two such amplitudes interfere, without the need for neutral B mixing. Asymmetry measurements in charged B decays and in self-tagged neutral B decays have the advantage of avoiding the price of flavor tagging. However their theoretical interpretation in terms of the weak phase γ involves an unknown strong phase and an unknown ratio of two interfering hadronic matrix elements. A few methods were proposed to overcome this difficulty, often by measuring not only the asymmetry, but also the rates of certain related processes which provide information on these matrix elements. In the next few sections we describe several ways of measuring γ in four classes of processes, discussing in each case both the theoretical uncertainties and the experimental limitations.

2 $B \rightarrow DK^{(*)}$

A simple idea for measuring γ in $B^\pm \rightarrow DK^\pm$ was proposed some time ago¹⁴. Neglecting very small CP violation in $D^0 - \bar{D}^0$ mixing, one can write a triangle amplitude relation

$$\sqrt{2}A(B^+ \rightarrow D_1^0 K^+) = A(B^+ \rightarrow D^0 K^+) + A(B^+ \rightarrow \bar{D}^0 K^+) . \quad (2)$$

D_1^0 is an even CP-eigenstate, decaying, for instance, to $\pi^+\pi^-$, while D^0 and \bar{D}^0 are two opposite flavor states. The two amplitudes on the right-hand-side contain CKM factors $V_{ub}^*V_{cs}$ and $V_{cb}^*V_{us}$, both of which are $\mathcal{O}(\lambda^3)$ ¹⁵, and have a relative weak phase γ . If the two amplitudes were of about equal magnitudes, then the triangle relation and its charge-conjugate would permit a determination of $\sin \gamma$ within certain discrete ambiguities.

At a closer examination one observes, however, that the first amplitude is suppressed relative to the second one by a CKM factor, $|V_{ub}^*V_{cs}|/|V_{cb}^*V_{us}| \approx 0.4$, and by a color-suppression factor of about 0.25. [This prediction comes from evidence for color-suppression in $B \rightarrow \bar{D}\pi$ ¹⁶]. An order of magnitude suppression of this amplitude relative to the measured amplitude¹⁷ of $B^+ \rightarrow \bar{D}^0 K^+$, causes serious experimental difficulties in tagging the flavor of D^0 , through decays such as $D^0 \rightarrow K^-\pi^+$. The very rare decay $B^+ \rightarrow D^0 K^+ \rightarrow (K^-\pi^+)K^+$ interferes strongly with the doubly Cabibbo-suppressed decay of \bar{D}^0 in $B^+ \rightarrow \bar{D}^0 K^+ \rightarrow (K^-\pi^+)K^+$.

Two ways were proposed for partially evading this difficulty by considering only very rare decays, typically with branching ratios of order 10^{-7} . Aside from the small rates, both methods have other limitations.

- Study only color-suppressed decays¹⁸ $B^0 \rightarrow D^0(\bar{D}^0)K^{*0}$, where the flavors of neutral D and K^* are determined through $D^0 \rightarrow K^-\pi^+$, $K^{*0} \rightarrow K^+\pi^-$. Here the interference between Cabibbo-favored and Cabibbo-suppressed neutral D decays is much weaker. Still, such interference occurs and prohibits a precise determination of γ .
- Study $B^+ \rightarrow D^0(\bar{D}^0)K^+ \rightarrow fK^+$, with two neutral D decay modes, such as $f = K^-\pi^+$, $K^-\rho^+$. In this case the two interfering amplitudes have comparable magnitudes, one being color-suppressed in B decay and the other being doubly-Cabibbo-suppressed in D decay. Measurement of the rates of these two processes and their charge-conjugates would permit a determination of γ ¹⁹, provided that the two doubly Cabibbo-suppressed D decay branching ratios were known. Uncertainties in these branching ratios prohibit an accurate determination of γ . An intrinsic uncertainty²⁰ follows from the difficulty of disentangling doubly Cabibbo-suppressed D^0 decays from $D^0 - \bar{D}^0$ mixing²¹. The mixing may be larger than conventionally estimated²², as anticipated recently from large resonance contributions in D^0 decays²³. Therefore, a precise measurement of γ using this method requires knowledge of the $D^0 - \bar{D}^0$ mixing parameters.

There are also variants²⁴, which combine the above two schemes^{14,19} based on CP and flavor states. In all cases, the very small branching ratios of the color-suppressed processes imply that such measurements cannot be carried out effectively in the first round of experiments at $e^+e^- B$ factories, and would have to wait for hadron collider experiments providing at least 10^9 B 's.

An interesting question remains²⁵: What can be learned by studying only the more abundant color-allowed $B^\pm \rightarrow DK^\pm$ decays which have larger branching ratios? Considers the charge-averaged ratio of rates for neutral D mesons decaying to an even (odd) CP state and for a color-allowed flavor state

$$R_i \equiv \frac{2[\Gamma(B^+ \rightarrow D_i K^+) + \Gamma(B^- \rightarrow D_i K^-)]}{\Gamma(B^+ \rightarrow \bar{D}^0 K^+) + \Gamma(B^- \rightarrow D^0 K^-)} \quad , \quad i = 1, 2 \quad . \quad (3)$$

One finds (neglecting the small $D_1^0 - D_2^0$ width-difference)

$$R_{1,2} = 1 + \bar{r}^2 \pm 2\bar{r} \cos \bar{\delta} \cos \gamma \quad , \quad (4)$$

where $A(B^+ \rightarrow D^0 K^+)/A(B^+ \rightarrow \bar{D}^0 K^+) \equiv \bar{r} \exp[i(\bar{\delta} + \gamma)]$. This leads to two general inequalities

$$\sin^2 \gamma \leq R_{1,2} \quad , \quad i = 1, 2 \quad , \quad (5)$$

one of which *must* imply a new constraint on γ , unless $\gamma = \pi/2$.

Assuming, for instance, $\bar{r} = 0.1$, $\bar{\delta} = 0$, $\gamma = 40^\circ$, one finds $R_2 = 0.85$. With 10^8 B^+B^- pairs, and using measured B and D decay branching ratios²⁶, one estimates an error²⁵ $R_2 = 0.85 \pm 0.05$. In this case, Eq.(5) excludes the range $73^\circ < \gamma < 107^\circ$ with 90% confidence level. Including measurements of the CP asymmetries in $B^\pm \rightarrow D_i K^\pm$ permit, in principle, a determination of γ (and not only bounds on the angle). This depends, of course, on the unknown strong phase $\bar{\delta}$. For further studies of related methods, see²⁷.

3 $B_s^0(t) \rightarrow D_s K$

The interference between the two λ^3 subprocesses $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ operates also in the time-dependent decay $B_s^0 \rightarrow D_s^- K^+$, leading to a $\sin(\Delta m_s t)$ term²⁸

$$\begin{aligned} \Gamma(B_s^0(t) \rightarrow D_s^- K^+) = e^{-\Gamma_s t} [& |A|^2 \cos^2\left(\frac{\Delta m_s t}{2}\right) + |\bar{A}|^2 \sin^2\left(\frac{\Delta m_s t}{2}\right) \\ & - |A\bar{A}| \sin(\delta + \gamma) \sin(\Delta m_s t)] . \end{aligned} \quad (6)$$

The two color-allowed amplitudes, corresponding to $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$, have magnitudes $|A|$ and $|\bar{A}|$, and involve relative strong and weak phases, δ and γ , respectively.

These four parameters describe in a similar way the time-dependence in the three processes in which the initial and/or final states are charge-conjugated, $\bar{B}_s^0(t) \rightarrow D_s^- K^+$, $B_s^0(t) \rightarrow D_s^+ K^-$, $\bar{B}_s^0(t) \rightarrow D_s^+ K^-$. Thus, measuring the time-dependent oscillations in these four processes, all of which require tagging the flavor of initial B_s^0 , permits a determination of γ ²⁹. It is obvious that, for an accurate measurement, one would also need to know the width-difference between the two B_s mass eigenstates, which was neglected in (6). Further studies and discussions of width-dependent effects can be found in³⁰.

4 $B \rightarrow PP$

We will consider B decays to two light pseudoscalars, $B \rightarrow PP$ where $P = \pi, K$, within the framework of flavor SU(3) symmetry^{31,32,33,34}. Final states involving η and η' can be studied similarly³⁵. Occasional reference to SU(3) breaking effects will be made. A more ambitious approach, relying on generalized factorization^{10,36,37,38}, has not yet been justified quantitatively to a satisfactory level.

The low energy effective weak Hamiltonian³⁹ governing $B \rightarrow PP$

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left(\sum_{q'=u,c} \lambda_{q'}^{(q)} [c_1 Q_1^{(q')} + c_2 Q_2^{(q')}] - \lambda_t^{(q)} \sum_{i=3}^{10} c_i Q_i^{(q)} \right), \quad (7)$$

where $\lambda_{q'}^{(q)} = V_{q'b}^* V_{q'q}$, consists of the sum of three types of four quark operators: two $(V-A)(V-A)$ current-current operators ($Q_{1,2}$), four QCD penguin operators ($Q_{3,4,5,6}$), and four electroweak penguin (EWP) operators ($Q_{7,8,9,10}$) with different chiral structures. The EWP operators with dominant Wilson coefficients, Q_9 and Q_{10} ($|c_{7,8}|/|c_9| \approx 0.04$), have both a $(V-A)(V-A)$ structure. All the four-quark operators, $(\bar{b}q_1)(\bar{q}_2q_3)$, can be decomposed into a sum of $\mathbf{\overline{15}}$, $\mathbf{6}$ and $\mathbf{\overline{3}}$ representations^{31,33,34}. The QCD penguin operators are pure $\mathbf{\overline{3}}$.

A simple proportionality relation was noted recently⁴⁰ to hold between the dominant EWP operators Q_9 and Q_{10} and the current-current (“tree”) operators Q_1 and Q_2 , both transforming as given SU(3) representations

$$\mathcal{H}_{EWP}^{(q)}(\mathbf{\overline{15}}) = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \frac{\lambda_t^{(q)}}{\lambda_u^{(q)}} \mathcal{H}_T^{(q)}(\mathbf{\overline{15}}), \quad (8)$$

$$\mathcal{H}_{EWP}^{(q)}(\mathbf{6}) = \frac{3}{2} \frac{c_9 - c_{10}}{c_1 - c_2} \frac{\lambda_t^{(q)}}{\lambda_u^{(q)}} \mathcal{H}_T^{(q)}(\mathbf{6}). \quad (9)$$

The superscripts $q = d, s$ denote strangeness-conserving and strangeness-changing transitions, respectively. The two ratios of Wilson coefficients are equal within 3%³⁹

$$\frac{c_9 + c_{10}}{c_1 + c_2} \approx \frac{c_9 - c_{10}}{c_1 - c_2} \approx -1.12\alpha. \quad (10)$$

As a consequence of (8) and (9), processes given by $\mathbf{\overline{15}}$ and $\mathbf{6}$ transitions contain EWP and current-current contributions which are proportional to each other. For instance, in the $\Delta S = 1$ pure $\mathbf{\overline{15}}$ transition, $B \rightarrow (K\pi)_{I=3/2}$, where $|I = 3/2\rangle = |K^0\pi^+\rangle + \sqrt{2}|K^+\pi^0\rangle$, the ratio of these amplitudes

$$\delta_{EW} = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \frac{|V_{tb}^* V_{ts}|}{|V_{ub}^* V_{us}|} = 0.65 \pm 0.15 \quad (11)$$

is of order one⁴¹, in spite of the fact that EWP amplitudes are higher order in electroweak couplings.

This simple result, obtained in the limit of flavor SU(3), was used⁴² in order to obtain a bound on γ in $B^\pm \rightarrow K\pi$. Expressing $B^+ \rightarrow K\pi$ in terms of reduced SU(3) amplitudes depicted in graphical form³³, one has⁴³

$$\begin{aligned} A(B^+ \rightarrow K^0\pi^+) &= \lambda_t^{(s)}[P + EW] + |\lambda_u^{(s)}|e^{i\gamma}[P_{uc} + A] , \\ \sqrt{2}A(B^+ \rightarrow K^+\pi^0) &= -\lambda_t^{(s)}[P + EW] - |\lambda_u^{(s)}|[(T + C)(e^{i\gamma} - \delta_{EW}) + P_{uc} + A] . \end{aligned} \quad (12)$$

The dominant QCD penguin amplitudes P in the two processes, carrying a weak phase $\text{Arg}\lambda_t^{(s)} = \pi$, are equal because of isospin. Unequal rates of the two processes would be evidence for interference with smaller amplitudes involving a different weak phase. Defining the charge-averaged ratio of rates⁴²

$$R_*^{-1} \equiv \frac{2[B(B^+ \rightarrow K^+\pi^0) + B(B^- \rightarrow K^-\pi^0)]}{B(B^+ \rightarrow K^0\pi^+) + B(B^- \rightarrow \bar{K}^0\pi^-)} , \quad (13)$$

one finds, to leading order in small quantities

$$R_*^{-1} = 1 - 2\epsilon \cos\phi(\cos\gamma - \delta_{EW}) + \mathcal{O}(\epsilon^2) + \mathcal{O}(\epsilon\epsilon_A) + \mathcal{O}(\epsilon_A^2) . \quad (14)$$

ϵ and ϵ_A are ratios of tree-to-penguin and rescattering⁴⁴-to-penguin amplitudes, respectively

$$\epsilon e^{i\phi} = \frac{|\lambda_u^{(s)}|}{|\lambda_t^{(s)}|} \frac{T + C}{P + EW} , \quad \epsilon_A = \frac{|\lambda_u^{(s)}[P_{uc} + A]|}{|\lambda_t^{(s)}[P + EW]|} . \quad (15)$$

The first ratio is given by^{8,45}

$$\epsilon = \sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} \frac{|A(B^+ \rightarrow \pi^0\pi^+)|}{|A(B^+ \rightarrow K^0\pi^+)|} = 0.21 \pm 0.05 , \quad (16)$$

while the second one is roughly^{8,46}

$$\epsilon_A \sim \lambda \frac{|A(B^+ \rightarrow \bar{K}^0 K^+)|}{|A(B^+ \rightarrow K^0\pi^+)|} < 0.12 \quad (17)$$

Neglecting second order terms, one obtains a bound⁴²

$$|\cos\gamma - \delta_{EW}| \geq \frac{|1 - R_*^{-1}(K\pi)|}{2\epsilon} , \quad (18)$$

which would provide useful information about γ if a value different from one were measured for R_*^{-1} . The present value, $R_{*\text{exp}}^{-1} = 1.27 \pm 0.48$, is consistent

with one. Further information about γ , applying also to the case $R_*^{-1} = 1$, can be obtained by measuring separately B^+ and B^- decay rates⁴⁷. The solution obtained for γ involves uncertainties due to SU(3) breaking in subdominant amplitudes and an uncertainty in $|V_{ub}/V_{cb}|$, both of which affect the value of δ_{EW} . Combined with errors in $\epsilon \propto |A(B^+ \rightarrow \pi^+\pi^0)/A(B^+ \rightarrow K^0\pi^+)|$, and in the rescattering parameter ϵ_A , the resulting uncertainty in γ is unlikely to be smaller than 20° ⁴⁷. For other ways of studying γ in $B \rightarrow K\pi$, see⁴⁸.

5 $B \rightarrow VP$

B mesons decays to a charmless vector meson (V) and a pseudoscalar meson (P) involve a larger number of SU(3) amplitudes^{49,50} than $B \rightarrow PP$. SU(3) relations between EWP and current-current amplitudes, following from (8) and (9), reduce considerably the number of independent amplitudes. In this section we describe briefly two applications⁴⁰.

5.1 $B^\pm \rightarrow \rho K$

Defining a charge-averaged ratio of rates for $B^\pm \rightarrow \rho K$

$$R_*^{-1}(\rho K) \equiv \frac{2[B(B^+ \rightarrow \rho^0 K^+) + B(B^- \rightarrow \rho^0 K^-)]}{B(B^+ \rightarrow \rho^+ K^0) + B(B^- \rightarrow \rho^- \bar{K}^0)}, \quad (19)$$

one obtains, with some analogy to (18), the bound

$$|\cos \gamma| \geq \frac{|1 - R_*^{-1}(\rho K)|}{2\epsilon_V} - \delta_{EW} \left(\frac{\epsilon_P}{\epsilon_V} \right), \quad (20)$$

where

$$\epsilon_V = \sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} \frac{|A(B^+ \rightarrow \rho^0 \pi^+)|}{|A(B^+ \rightarrow \rho^+ K^0)|}, \quad \epsilon_P = \sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_{K^*}}{f_\rho} \frac{|A(B^+ \rightarrow \rho^+ \pi^0)|}{|A(B^+ \rightarrow \rho^+ K^0)|}. \quad (21)$$

Although this constraint is weaker than (18), it shows that measuring charge-averaged ratios of rates, which differ from one, is of interest also for PV final states.

5.2 $B^0 \rightarrow K^{*\pm} \pi^\mp$ vs. $B^\pm \rightarrow \phi K^\pm$

Considering the amplitudes for $B^0 \rightarrow K^{*+} \pi^-$ and $B^+ \rightarrow \phi K^+$, both of which are expected to be QCD penguin dominated, and keeping only dominant and

subdominant terms, one finds

$$\begin{aligned} A(B^0 \rightarrow K^{*+}\pi^-) &= -\lambda_t^{(s)} P_P - \lambda_u^{(s)} T_P , \\ A(B^+ \rightarrow \phi K^+) &= \lambda_t^{(s)} [P_P + EW] , \end{aligned} \quad (22)$$

where the suffix P denotes the pseudoscalar which includes the spectator in the graphic SU(3) amplitudes⁴⁹. The EWP contribution is related to the tree amplitude by⁴⁰

$$\lambda_t^{(s)} EW = \frac{1}{3} \delta_{EW} |\lambda_u^{(s)}| T_P . \quad (23)$$

Defining the ratio

$$R = \frac{|A(B^0 \rightarrow K^{*+}\pi^-)|^2 + |A(\bar{B}^0 \rightarrow K^{*-}\pi^+)|^2}{|A(B^+ \rightarrow \phi K^+)|^2 + |A(B^- \rightarrow \phi K^-)|^2} , \quad (24)$$

one finds

$$R = \frac{1 + r^2 - 2r \cos \delta \cos \gamma}{1 + (\delta_{EW}/3)^2 r^2 - (2/3) \delta_{EW} r \cos \delta} , \quad (25)$$

where

$$r e^{i\delta} = \frac{|\lambda_u^{(s)}| T_P}{|\lambda_t^{(s)}| P_P} . \quad (26)$$

Present 90% confidence level limits⁸, $\mathcal{B}(B^0 \rightarrow K^{*\pm}\pi^\mp) > 12 \times 10^{-6}$ and $\mathcal{B}(B^\pm \rightarrow \phi K^\pm) < 5.9 \times 10^{-6}$, imply $R > 2$, which is evidence for a nonzero contribution of T_P . In order to use this inequality for information about γ , one must include some input about r and δ . A reasonable assumption, supported both by perturbative³⁶ and statistical⁵¹ calculations, is that δ does not exceed 90° . A very conservative assumption about r is $r \leq 1$. Making these two assumptions, one finds [for δ_{EW} we will use the range (11)]

$$\cos \gamma - \frac{2}{3} \delta_{EW} < \frac{-1 + r^2 [1 - 2(\delta_{EW}/3)^2]}{2r} . \quad (27)$$

This implies $\gamma > 62^\circ$ for $r = 1$, and $\gamma > 105^\circ$ for $r = 0.5$. Indirect evidence⁵⁰ exists already for $r < 0.55$. Direct information on r will be obtained from future rate measurements of $B^+ \rightarrow K^{*0}\pi^+$ and $B^+ \rightarrow \rho^+\pi^0$ or $B^0 \rightarrow \rho^+\pi^-$.

These bounds neglect smaller terms in the amplitudes, primarily the color-suppressed tree amplitude. If this contribution is 10% (20%) of the color-favored tree amplitude T_P ³⁶, then the limits move up or down by about 5° (10°). The bounds also assume [by SU(3)] equal QCD penguin contributions in the two processes. The constraint becomes stronger if the penguin amplitude in $B^+ \rightarrow \phi K^+$ is larger than in $B^0 \rightarrow K^{*+}\pi^-$, as predicted by factorization⁵². However, the constraint may become weaker if SU(3) breaking in penguin amplitudes is not described by factorization.

6 Conclusion

We discussed several ways of determining the weak phase γ . The first two schemes, based on $B \rightarrow DK$ and $B_s \rightarrow D_s K$, seem at first sight to involve no hadronic uncertainties. However, at a closer look, they require taking care of smaller effects, such as $D^0 - \bar{D}^0$ mixing and the width-difference in the B_s^0 system. Methods based on charmless decays to two pseudoscalars, and decays to a pseudoscalar and a vector meson, involve dynamical hadronic parameters, such as those describing SU(3) breaking and rescattering amplitudes. Some of these quantities are under control, and others should be studied through a dialogue between theory and experiments.

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References

1. For a recent review, see M. Gronau, hep-ph/9908343, to appear in the Proceedings of the 1999 Chicago Conference on Kaon Physics, Chicago, IL, June 1999, J. L. Rosner and B. Winstein, eds. (University of Chicago Press, 2000). For an earlier comprehensive study, see *The BaBar Physics Book*, P. F. Harrison and H. R. Quinn, eds. (SLAC Report SLAC-R-504, 1998).
2. A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. **193**, 85 (1981).
3. M. Gronau, Phys. Rev. Lett. **63**, 1451 (1989).
4. D. London and R. D. Peccei, Phys. Lett. B **223**, 257 (1989).
5. W. Trischuk, these proceedings; CDF Collaboration, hep-ex/9909003.
6. R. Sinha, these proceedings; D. London, N. Sinha and R. Sinha, Phys. Rev. D **60**, 074020 (1999).
7. A. Dighe, M. Gronau and J. L. Rosner, Phys. Rev. Lett. **79**, 4333 (1997); M. Gronau and J. L. Rosner, hep-ph/9909478, Phys. Rev. D. **61**, 0540xx (2000).
8. J. Smith, these proceedings; CLEO Collaboration, CLEO CONF 99-13, CLEO CONF 99-14.

9. F. DeJongh and P. Sphicas, Phys. Rev. D **53**, 4930 (1996); P. S. Marrocchesi and N. Paver, Int. J. Mod. Phys. A **13**, 251 (1998); J. Charles, Phys. Rev. D **59**, 054007 (1999).
10. N. Deshpande, these proceedings; K. Agashe and N. G. Deshpande, hep-ph/9909298.
11. M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
12. H. J. Lipkin, Y. Nir, H. R. Quinn and A. Snyder, Phys. Rev. D **44**, 1454 (1991); M. Gronau, Phys. Lett. B **265**, 389 (1991); H. R. Quinn and A. Snyder, Phys. Rev. D **48**, 2139 (1993).
13. I. Dunietz, Proceedings of the Workshop on B Physics at Hadron Accelerators, Snowmass, CO, 1993, p. 83; D. Pirjol, Phys. Rev. D **60**, 54020 (1999); R. Fleischer, Phys. Lett. B **459**, 306 (1999).
14. M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
15. L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
16. T. E. Browder, K. Honscheid and D. Pedrini, Ann. Rev. Nucl. Part. Sci. **46**, 395 (1997).
17. CLEO Collaboration, M. Athanas *et al.*, Phys. Rev. Lett. **80**, 5493 (1998).
18. M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991); I. Dunietz, Phys. Lett. B **270**, 75 (1991).
19. D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997).
20. J. P. Silva and A. Soffer, hep-ph/9912242.
21. D. Asner, these proceedings; CLEO Collaboration, CLEO CONF 99-17.
22. L. Wolfenstein, Phys. Lett. B **164**, 170 (1985); J. Donoghue, E. Golowich, B. Holstein and J. Trampetic, Phys. Rev. D **33**, 179 (1986); T. A. Kaeding, Phys. Lett. B **357**, 151 (1995).
23. M. Gronau, Phys. Rev. Lett. **83**, 4005 (1999).
24. A. Soffer, Phys. Rev. D **60**, 054032 (1999); S. L. Stone, private communication.
25. M. Gronau, Phys. Rev. D **58**, 037301 (1998).
26. Particle Data Group, C. Caso, *et al.*, Zeit. Phys. C **3**, 1 (1998).
27. Z.-Z. Xing, Phys. Rev. D **58**, 093005 (1998); M. Gronau and J. L. Rosner, Phys. Lett. B **439**, 171 (1998); J.-H. Jang and P. Ko, Phys. Rev. D **58**, 111302 (1998); N. Sinha and R. Sinha Phys. Rev. Lett. **80**, 3706 (1998); B. Kayser and D. London, hep-ph/9909561.
28. M. Gronau, Phys. Lett. B **23**, 479 (1989).
29. R. Aleksan, I. Dunietz, B. Kayser and F. Le Diberder, Nucl. Phys. **361**, 141 (1991).
30. R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Zeit. Phys. C **67**, 251 (1995); I. Dunietz, Phys. Rev. D **52**, 3048 (1995); R.

- Fleischer and I. Dunietz, Phys. Rev. D **55**, 259 (1997).
31. D. Zeppenfeld, Zeit. Phys. C **8**, 77 (1981).
 32. L. L. Chau *et al.*, Phys. Rev. D **43**, 2176 (1991).
 33. M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994).
 34. B. Grinstein and R.F. Lebed, Phys. Rev. D **53**, 6344 (1996).
 35. A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Lett. B **367**, 357 (1996); **377**, 325(E) (1996); Phys. Rev. D **54**, 3309 (1996); Phys. Rev. Lett. **79**, 4333 (1997).
 36. M. Beneke, G. Buchalla, M. Neubert, and C. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
 37. H.-Y. Cheng, these proceedings; H.-Y. Cheng and K.-C. Yang, hep-ph/9910291.
 38. H.-N. Li, these proceedings, and hep-ph/9903323, DPF99, UCLA, 1999.
 39. G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
 40. M. Gronau, hep-ph/9911429.
 41. R. Fleischer, Zeit. Phys. C **62**, 81 (1994); N. G. Deshpande and X.-G. He, Phys. Lett. B **336**, 471 (1994).
 42. M. Neubert and J. L. Rosner, Phys. Lett. B **441**, 403 (1998).
 43. M. Gronau, D. Pirjol, and T.-M. Yan, Phys. Rev. D **60**, 034021 (1999).
 44. For a definition of rescattering in terms of graphical SU(3) amplitudes, see M. Gronau and J. L. Rosner, Phys. Rev. D **58**, 113005 (1998).
 45. M. Gronau, J. L. Rosner, and D. London, Phys. Rev. Lett. **73**, 21 (1994).
 46. A. Falk, A. L. Kagan, Y. Nir, and A. A. Petrov, Phys. Rev. D **57**, 6843 (1998).
 47. M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81**, 5076 (1998); M. Neubert, JHEP **9902**, 014 (1999); M. Gronau and D. Pirjol, Phys. Rev. D **61**, 013005 (2000).
 48. R. Fleischer and T. Mannel, Phys. Rev. D **57**, 2752 (1998); M. Gronau and J. L. Rosner, Phys. Rev. D **57**, 6843 (1998); A. Buras and R. Fleischer, Euro. Phys. J. C **11**, 93 (1999).
 49. A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Rev. D **57**, 1783 (1998).
 50. M. Gronau and J. L. Rosner, Ref. 7.
 51. M. Suzuki, these proceedings; M Suzuki and L. Wolfenstein, Phys. Rev. D **60**, 074019 (1999).
 52. C.-D. Lu, these proceedings; A. Ali, G. Kramer, and C.-D. Lu, Phys. Rev. D **58**, 094009 (1998).